Int. J. Heat Mass Transfer. Vol. 15, pp. 2519–2534. Pergamon Press 1972. Printed in Great Britain

RADIATIVE HEAT TRANSFER BY FLOWING MULTIPHASE MEDIUM–PART I. AN ANALYSIS ON HEAT TRANSFER OF LAMINAR FLOW BETWEEN PARALLEL FLAT PLATES

RYOZO ECHIGO and SHU HASEGAWA

Department of Nuclear Engineering, Kyushu University, Fukuoka, Japan

(Received 31 *July* 1971*)*

Abstract—The multiphase flow of gaseous suspensions of fine particles furnishes high heat transfer characteristics at high and/or extremely high temperatures and at high heat fluxes due to the radiative transfer from heat source to suspensions. The phaseshift of particulate medium improves the overall heat transfer remarkably and from the practical viewpoint there exists important relevance pertinent to the industrial applications.

It is worth having a closer look at the behaviors of the suspensions and the heat transfer mechanism in flowing multiphase media so that the discussions are held concerning the foregoing media in some details.

An analysis is carried out on the laminar flow between parallel plates by taking into account of thermal radiation and the results illustrate the temperature profiles of fluid and dispersed phase, respectively, and the heat transfer characteristics for the wide ranges of dimensionless parameters such as conduction~radiation interaction parameter, loading ratio of particles, optical depth of duct, heat transfer between the two phases and so forth. Reference to the temperature profiles reveals the facts that while the temperature gradient in the vicinity of the heating surface increases due to the presence of particulate phase, the cup-mixing mean temperature is raised appreciably by thermal radiation through the dispersed medium. In consequence, the contributions of suspensions on heat transfer are drastic, particularly in high temperature cases. Alternatively the correlations between the foregoing dimensionless parameters are also examined in current study.

NOMENCLATURE

A_p, surface area of single particle;

B_0, Boltzmann number $(=4n^2\sigma T^3_w/\rho_f c_f \bar{u}_f)$;

c_f, c_p, specific heat of fluid and particulate phase;

d_e, equivalent diameter of flow channel;

d_p, diameter of particle;

E, Euler's constant $(=0{\cdot}57721566\cdots)$;

$E_{bb}(\tau)$, radiation energy emitted by black-body at τ;

$E_n(\tau)$, function of exponential integral $(=\int_0^1 \mu^{n-2}\exp(-\tau/\mu)\,d\mu)$;

h_p, heat transfer coefficient between particulate and fluid phase;

h_x, local heat transfer coefficient of duct;

k_f, thermal conductivity of fluid;

M, dimensionless parameter defined by equation (20);

N_R, interaction parameter of conduction to radiation;

Nu_d, Nusselt number of heat transfer between particle and fluid;

Nu_x, local Nusselt number defined by equation (25);

$Nu_{x,\,con}$, $\check{N}u_{x,\,con}$, local Nusselt number of convection defined by equations (33) and (35);

$Nu_{x,\,rad}$, local Nusselt number of radiative transfer defined by equation (34);

n, refractivity;

n_p, particle density (number of particles in unit volume);

Pr, Prandtl number;
q_{con}, convective heat flux defined by equation (26);
q^R, radiative heat flux defined by equation (27);
q^{total} total heat flux defined by equation (24);
$R(0)$, $R(\tau)$, radiosity;
Re, Reynolds number;
T_f, T_p, temperature of fluid and particle;
T_m, cup-mixing mean temperature of multiphase medium;
T_0, temperature of multiphase medium at entrance of duct;
T_w, wall temperature;
U, U_j, dimensionless velocity $(u/\bar{u})$;
u, $\bar{u}$, velocity, mean velocity;
u_f, u_p, velocities of fluid and particle;
V_p, volume of single particle;
X, ΔX, dimensionless independent variable and its finite difference, equation (12);
Y, ΔY, dimensionless independent variable and its finite difference, equation (12);
x, y, coordinate system;
y_0, distance between parallel plates.

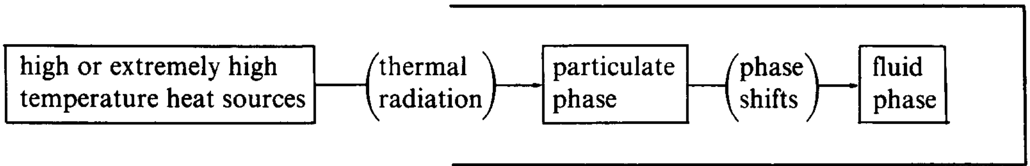
Greek symbols
Γ thermal loading ratio of particle defined by equation (15);
γ, loading ratio of particle defined by equation (15);
ε_p, emissivity of particle;
ε_w, wall emissivity;
Θ, dimensionless temperature difference θ–θ_o normalized by $(\theta_w - \theta_0)$;
θ^f, θ^p, dimensionless temperatures of fluid and particle;
θ_m, dimensionless cup-mixing mean temperature of multiphase in equation (32);
$\theta_{m,f}$, $\theta_{m,p}$, dimensionless cup-mixing mean temperatures of fluid and particles in equations (30) and (31);
θ_0, dimensionless temperature at entrance $(= T_0/T_w)$;
θ_w, dimensionless temperature of wall;
$\tilde{\theta}$, polar angle;
κ, absorption coefficient of particulate medium defined by equation (4);
μ, $= \cos \tilde{\theta}$;
ρ_{dp}, apparent density of dispersed (particulate) medium;
ρ_f, ρ_p, densities of fluid and particle;
σ, Stefan–Boltzmann constant;
τ, optical depth;
τ_0, optical thickness;
ϕ, azimuthal angle;
Ω, solid angle.

Super- and subscripts
dp, dispersed phase;
f, fluid;
0, entrance;
p, particle
R, radiation;
w, wall;
$'$, dummy variable.

1. INTRODUCTION

THE MULTIPHASE medium is tentatively defined here as the mixtures of the dispersed phase, which consists of fine particles of solid and/or liquid, and the carrier medium of fluid phase with an exception that both phases are in liquid. For the heat transfer studies at high temperature, however, the gaseous carrier is preferable. A considerable number of studies [1–3] have been issued in the past due to the facts that the multiphase bears heat transfer with high heat flux by virtue of the large heat capacity of particulate phase and that it might facilitate to realise high heat transfer characteristics by the behaviours of particles. An exposition on this subject appears in the text [4] by Soo.

A detailed understanding of the heat transfer mechanisms relating to the behaviours of the multiphase medium has not been enunciated thereafter, particularly in the system at high temperature where the thermal radiation should be taken into account. Further it must be stressed that the multiphase medium exhibits a prominent feature of heat transfer in the field of thermal radiation and it is not exaggeration to mention that there exist no other media comparable to multiphase at high or extremely high temperature heat transfer. The present study includes the phase shifts (melting, evaporation and sublimation) of the particulate clouds, in the media so as to facilitate the overall heat transfer characteristics, eliminating the rate process of heat transfer between the two phases. The heat transfer mechanism is expressed briefly as follows:



(laminar or turbulent) and configuration of duct (curved channel, variation of cross sectional area along the flow, etc.) and also such factors pertinent to dispersion as the mean size and size distribution, density, melting and boiling points, specific heat and the loading ratio (mass flow rate of particles to that of fluid). When the electromagnetic forces exert upon the particles with electric charge or the sonic excitations upon the multiphase flow field the heat transfer characteristics might be influenced to a certain extent. Furthermore the radiative properties of multiphase such as absorptivity, reflectivity, transmissivity and refractivity might affect the heat transfer drastically.

This system can be applied to various kinds of practical facilities of heat transfer at high or extremely high temperature and at high heat flux such as gaseous core nuclear reactors, plasma and high temperature heat exchanger. It is the objective of this study to determine the heat transfer characteristics for the flowing multiphase medium in the field of thermal radiation.

2. SOME DISCUSSIONS ON HEAT TRANSFER BY MULTIPHASE MEDIUM

Although the multiphase system takes priority over other heat transfer media regarding the high temperature and high flux heat transfer, the elaborate knowledge on the phenomena seems to be not enough. Consequently it is available to summarize the problems embodied in this system and to discuss them briefly, from the speculative and practical viewpoints.

The major concerns are the behaviours of dispersed media upon such hydrodynamic factors as flow velocity, viscous force, flow regime

(a) *Loading ratio*

It is favourable for the engineering applications that the pressure drop induced by the flow of gaseous suspensions with loading ratio of 0(1) is almost the same as that of single phase flow [5]. Excepting the contribution of thermal radiation the heat transfer characteristics of the multiphase media increase with loading ratio [6] because the entrance length is approximately proportional to the loading ratio. The experimental results [7] of loading ratio up to $0(10^2)$ illustrate that the heat transfer coefficient becomes two or three orders of mangitude larger compared with that of single phase flow, although the details of experimental facility is not described. Qualitatively the relative velocity between dispersion particles and carrier fluid is affected by the loading ratio and the ultimate form with extremely large loading ratio in this system results in the heat transfer problems in packed beds. While the heat transfer characteristics by the multiphase flow is generally improved as the loading ratio γ increases, but in some cases without thermal radiation the cup-mixing mean temperature of fluid may be lower than that of single phase with the absence of thermal radiation because of the fact that approximately $1/(1 + \gamma)$ fraction of total amount of heat is transferred to the carrier fluid. The elaborate exposition on the friction factor and heat transfer coefficient of multiphase flow for a wide range of loading ratio might have not been elucidated.

(b) *Size and size distribution of particles*

The optical thickness of the flow duct is inversely proportional to the mean size of particles under the constant loading ratio, which leads to better radiative transfer due to the increased absorption of the medium for thermal radiation. The handling of particles with the diameter less than $10\,\mu$, however, becomes cumbersome for the practical purposes. Alternatively when the size is large or the excursion from the uniform size postulation (size distribution) is large the temperature distributions within the particles have to be taken into consideration. In turbulent flow regime a certain size of particles suppresses the turbulence of flow and, in consequence, suffers low heat transfer characteristics [8]. The counter-effects, one is decrease of heat transfer coefficient due to the depression of turbulence and the other is increment due to the presence of dispersion, have not systematically been examined so far.

(c) *Heat transfer between dispersion and carrier fluid, and turbulent Prandtl number of multiphase media*

Postulating that the dispersed media are spherical in shape and that the relative velocity of particle to fluid is zero, the Nusselt number defined by $Nu_d \equiv h_p d_p / k_f$ asymptotes to 2, which prescribes the heat transfer between two phases. Foregoing relative velocity is, however, inherent to multiphase flow owing to the large density of particulate media irrespective of the flow regime (laminar or turbulent). Even in laminar flow the heat transfer between two phases might be related to the particle size, loading ratio, Reynolds number, etc. In either case of comparatively large size of particle or of high temperature of the heating surface the temperature of particle is increased appreciably by thermal radiation and the temperature (enthalpy) difference of the respective medium will be drastic, wherein the heat transfer to fluid via thermal radiation is not efficient, in other words, the heat transfer between two phases is rating the entire processes of heat transfer from heat source to fluid. In turbulent flow, the evaluation of Nu_d being difficult, the heat transfer between two phases substantially increases so that the overall heat transfer characteristics to fluid is promoted. In an extremely high temperature system, however, the temperature difference between both phases might be prominent and the heat transfer from particles to fluid will be a controlling process again.

In order to eliminate such a rate process it will be plausible to introduce the dispersed particles with comparatively low melting and boiling points which yield phase shifts while travelling in duct. It will also be available for this purpose that the body forces induced by sonic and electromagnetic fields (when the particles have electric charge) exert upon dispersed media. In this connection the turbulent Prandtl number $\varepsilon_M/\varepsilon_H$ of multiphase medium has been examined extensively for evaluating the heat transfer processes but further analyses have to be subsequently performed in order to gain insight into the elaborate behaviour of multiphase flow.

(d) *Radiative properties of dispersed and fluid phases*

There are two cases of formulation of the problem when the carrier fluid is gaseous state. One is the transparent gas for thermal radiation (inactive for infrared) such as nitrogen, hydrogen, oxygen, helium which are of importance in engineering applications. Particularly the helium has been and will be used frequently for practical applications because of its stable situation at high temperature. Herein the thermal radiation neither affords the improvement of heat transfer at high temperature without the cloud of particles because of its transparency nor, even with particles, contributes to the heat exchange between two phases above convection.

Carbon dioxide and water vapour belong to the other case, in which these gases can absorb and emit the thermal radiation (active for infrared). In this case the heat transfer mechanism becomes rather difficult, that is, for the energy transport to the fluid in the central part of duct there exist several modes of transfer, namely, the convective transfer through fluid layers, the convection from particulate phase, which absorbs thermal radiation, and direct radiative transfer from the heating surface. Among these modes of energy transfer there should be a crucial interaction which seems to be an essential feature in this problem.

The absorption coefficients of infrared active gases are, in general, small and then it is not easy to realise a drastic increase of the absoptivity for radiation by pressurizing the entire loop of circulation. Consequently even for infrared active gases the additional effect of the dispersed medium on the radiative transfer might be remarkable and the opacity of carrier fluid for radiation improves the heat transfer coefficient between two phases to a certain extent.

As another feature in multiphase flow system it is feasible to alter the optical thickness of duct with the variation of loading ratio and the mean diameter of particles which yield a great increase in radiative heat transfer characteristics where one needs not to change the equivalent diameter of duct and the pressure of flowing medium. As outlined above although the heat transfer by the multiphase media involves many fundamental and technical problems it would be a plausible mean for high temperature and high flux heat transfer.

3. THEORETICAL ANALYSIS

3.1 *Description of the problem*

The analysis is concerned with the heat transfer of fully developed laminar flow of the multiphase medium between parallel flat plates as shown in Fig. 1 and based on the following

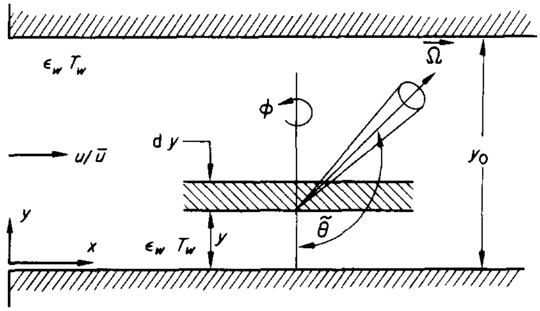


FIG. 1. Physical model and coordinate system.

assumptions: (i) The surfaces of the duct are considered to be isothermal, diffuse reflectors and emitters and grey ($\varepsilon_{w\lambda}$ = const.). (ii) The flows of both carrier and dispersed phases are laminar and the velocity profiles of both phases are parabolic where the flowing situation of fluid is not influenced by the presence of particles. (iii) The physical properties of the multiphase (ρ, c, μ, κ etc.) are constant. (iv) The dispersed medium is assumed to be fine spherical particles with uniform size and dispersed homogeneously which can absorb and emit thermal radiation in local thermodynamic equilibrium but cannot scatter. (v) The thermal diffusivity of particles is large and particle size is small so that the temperature within each particle is considered to be uniform. (vi) The dispersed phase is assumed to be continuum for thermal radiation and the fluid phase is transparent for radiation. (vii) The energy transferred by the collisions between particles is regarded to be negligible compared with that by conductive, convective and radiative transports. (viii) Neither the agglomerations of particulate phase nor the chemical reactions in multiphase are taken into account.

3.2 *Formulation of the basic equations and analytical method*

The basic equations governing the temperature profiles for particulate and fluid phase are in the following forms:

$$\rho_{dp}c_p u_p \frac{\partial T_p}{\partial x} = -n_p h_p A_p(T_p - T_f) - \frac{\partial q^R}{\partial y} \quad (1)$$

$$\rho_f c_f u_f \frac{\partial T_f}{\partial x} = k_f \frac{\partial^2 T_f}{\partial y^2} + n_p h_p A_p(T_p - T_f) \quad (2)$$

where subscripts p and f denote particulate and fluid medium respectively and the dispersed quantity is designated by dp. Therefore, ρ_{dp} means the apparent density of dispersed phase which is correlated to the density of particulate substance ρ_p

$$\rho_{dp} = \rho_p n_p V_p. \quad (3)$$

Postulating that the dispersed phase is continuum for the thermal radiation, the absorption coefficient is derived from a simple geometrical consideration concerning the particulate character of dispersed medium as follows:

$$\kappa = \pi(d_p/2)^2 n_p \varepsilon_p. \quad (4)$$

The optical length τ, the optical depth of a duct τ_0 and the exponential integral are defined in the following

$$\tau = \int_0^y \kappa \, \mathrm{d}y \quad (5)$$

$$\tau_0 = \int_0^{y_0} \kappa \, \mathrm{d}y \quad (6)$$

$$E_n(\tau) = \int_0^1 \mu^{n-2} \exp(-\tau/\mu)\, \mathrm{d}\mu \quad (\mu = \cos\tilde{\theta}). \quad (7)$$

Then the radiative heat flux $q^R(\tau)$ in equation (1) is in the form

$$q^R(\tau) = 2\Big[R(0)E_3(\tau) + \int_0^\tau E_{bb}(\tau')E_2(\tau - \tau')\,\mathrm{d}\tau' - R(\tau_0)E_3(\tau_0 - \tau) - \int_\tau^{\tau_0} E_{bb}(\tau')E_2(\tau' - \tau)\,\mathrm{d}\tau'\Big] \quad (8)$$

where primes denote the dummy variable and the radiosities $R(0)$ and $R(\tau_0)$ are equal owing to the constant temperature T_w of the two walls

$$R(0)/4\kappa\sigma T_w^4 = R(\tau_0)/4\kappa\sigma T_w^4 = \Big\{\varepsilon_w\theta_w^4 + 2(1 - \varepsilon_w)\int_0^{\tau_0} \theta^4(\tau)E_2(\tau_0 - \tau)\,\mathrm{d}\tau\Big\}\Big/\Big\{1 - 2(1 - \varepsilon_w)E_3(\tau)\Big\} \quad (9)$$

Here $\theta = T/T_w$ is used. The boundary conditions are

$$\left.\begin{aligned} &y = 0(\tau = 0), \\ &y = y_0(\tau = \tau_0); \quad T_f = T_p = T_w(\theta^f = \theta^p = 1) \\ &x = 0; \quad T_f = T_p = T_0(\theta^f = \theta^p = \theta_o). \end{aligned}\right\} \quad (10)$$

The discontinuity of the dispersed medium temperature at the wall has to be examined with an aid of the radiation slip [9] in a strict sense but the boundary conditions of equation (10), which is tentatively used, reduce the mathematical difficulty. It is further assumed that the velocity profiles for both phases are the same over the entire range of duct, that is

$$U = u_p/\bar{u}_p = u_f/\bar{u}_f = 6[(y/y_0) - (y/y_0)^2]. \quad (11)$$

This does not, however, necessitate the equality of the mean velocities of $\bar{u}_p$ and $\bar{u}_f$.

Multiplying the equations (1) and (2) by a factor $(y_0^2/k_f T_w)$ and transforming the variables defined as

$$\left.\begin{aligned} X &= (x/y_0)/RePr \\ Y &= y/y_0 = \tau/\tau_0 \end{aligned}\right\} \quad (12)$$

one can get the basic equations in dimensionless form by simple mathematical manipulations.

$$\Gamma U \frac{\partial \theta^p}{\partial X} = -Nu_a \frac{4\tau_0^2}{\kappa d_p \varepsilon_p}(\theta^p - \theta^f) + \frac{2\tau_0^2}{N_R}\Big\{\int_\tau^{\tau_0} \theta^{p^3}(\tau')\frac{\mathrm{d}\theta^p(\tau')}{\mathrm{d}\tau'}E_2(\tau' - \tau)\,\mathrm{d}\tau' - \int_0^\tau \theta^{p^3}(\tau')\frac{\mathrm{d}\theta^p(\tau')}{\mathrm{d}\tau'}E_2(\tau - \tau')\,\mathrm{d}\tau'\Big\} \quad (13)$$

$$U\frac{\partial \theta^f}{\partial X} = \frac{\partial^2 \theta^f}{\partial Y^2} + Nu_d \frac{4\tau_0^2}{\kappa d_p \varepsilon_p}(\theta^p - \theta^f) \quad (14)$$

where

$$\Gamma = \frac{\rho_{dp}\bar{u}_p c_p}{\rho_f \bar{u}_f c_f} = \gamma \frac{c_p}{c_f} \quad \left(\gamma = \frac{\rho_{dp}\bar{u}_p}{\rho_f \bar{u}_f}\right) \quad (15)$$

$$Nu_d = h_p d_p / k_f. \quad (16)$$

In equation (15) while γ is the "conventional" loading ratio prevalently used in multiphase flow, Γ means the ratio of heat capacity flow rate of two phases and this quantity is rather convenient dimensionless parameter in heat transfer of multiphase flow and called here "thermal" loading ratio of particles. Alternatively Nu_d in equation (16) is defined as Nusselt number for the heat transfer between particles and surrounding fluid which is influenced by many factors such as flow regime (laminar or turbulent), the presence of phase shifts and body forces exerted on particles and so forth. Nu_d tends to 2 asymptotically as the relative velocity between particles and fluid diminishes without the phase changes and the body forces.

Equations (13) and (14) constitute simultaneous integrodifferential equations with high order of nonlinearity and it seems to be formidable to get an analytical solution. Accordingly

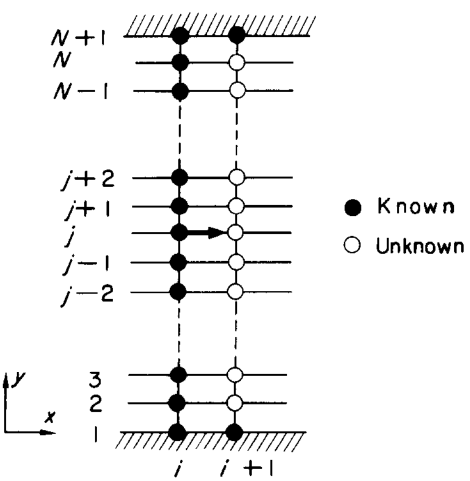


FIG. 2. Lattice of finite difference.

the numerical solution by a finite difference method might be pertinent to this problem. An examination of equation (14) leads to a tractable procedure in order to determine the lattice of finite differences along X and Y axes because of the parabolic form of equation (14); however, there may exist an uncertainty for equation (13). If the integral terms in equation (13) are used for iterative evaluation one may regard the foregoing equation as a modified form of linear differential equation in parabolic type. In consequence, based on the finite difference technique of implicit method, as illustrated in Fig. 2, the differentiations with respect to X, Y (or τ') in equations (13) and (14) can be replaced by

$$\left.\begin{aligned} &\frac{\partial \theta^p}{\partial X} = \frac{\theta^p_{i+1,j} - \theta^p_{i,j}}{\Delta X} \quad \frac{\partial \theta^f}{\partial X} = \frac{\theta^f_{i+1,j} - \theta^f_{i,j}}{\Delta X} \\ &\tau_0 \frac{\partial \theta^p}{\partial \tau'} = \frac{1}{2}\frac{\theta^p_{i+1,k+1} - \theta^p_{i+1,k-1}}{\Delta Y} \\ &\frac{\partial^2 \theta^f}{\partial Y^2} = \frac{\theta^f_{i+1,j+1} + \theta^f_{i+1,j-1} - 2\theta^f_{i+1,j}}{(\Delta Y)^2} \end{aligned}\right\} \quad (17)$$

and the substitutions of equation (17) into equations (13) and (14) and the approximation of the integrals in equation (13) by trapezoidal formula yield

$$\begin{aligned} \Gamma U_j \frac{\theta^p_{i+1,j} - \theta^p_{i,j}}{\Delta X} = {} & -M(\theta^p_{i+1,j} - \theta^f_{i+1,j}) \\ & + \frac{\tau_0^2}{N_R}\left\{ \sum_{k=j+1}^{N} E_2(\tau_0[Y_k - Y_j]) \right. \\ & \theta^{p^3}_{i+1,k}(\theta^p_{i+1,k+1} - \theta^p_{i+1,k-1}) \\ & + E_2(\tau_0[Y_{N+1} - Y_j])\theta^{p^3}_{i+1,N+1}(\theta^p_{i+1,N+1} - \theta^p_{i+1,N}) \\ & - \sum_{k=2}^{j-1} E_2(\tau_0[Y_j - Y_k])\theta^{p^3}_{i+1,k}(\theta^p_{i+1,k+1} - \theta^p_{i+1,k-1}) \\ & \left. - E_2(\tau_0[Y_j - Y_1])\theta^{p^3}_{i+1,1}(\theta^p_{i+1,2} - \theta^p_{i+1,1}) \right\} \quad (18) \end{aligned}$$

$$\begin{aligned} U_j \frac{\theta^f_{i+1,j} - \theta^f_{i,j}}{\Delta X} = {} & \frac{\theta^f_{i+1,j+1} + \theta^f_{i+1,j-1} - 2\theta^f_{i+1,j}}{(\Delta Y)^2} \\ & + M(\theta^p_{i+1,j} - \theta^f_{i+1,j}) \quad (19) \end{aligned}$$

where

$$M = Nu_d \frac{n_p A_p y_0^2}{d_p} = Nu_d \frac{4\tau_0^2}{\kappa d_p \varepsilon_p}. \quad (20)$$

As the temperatures at $j = 1, N+1$ are known from the boundary conditions the number of unknown temperature designated by the symbol ○ in Fig. 2 is $2(N-1)$ ($\theta^p_{i+1,j}$, $\theta^f_{i+1,j}$ for $j = 2$–N) and these temperatures can be obtained from the known temperatures of $N+3$ designated by the symbol ●. For the iterative procedure the temperatures of $\theta^p_{i+1,j}$ appeared in {} of equation (18) are, at first, approximated by $\theta^p_{i,j}$ and thereafter the derived solutions substitute the foregoing $\theta^p_{i+1,j}$ repeatedly until the desired convergence is satisfied. Once the convergence of iterative solution has been established, the progressive calculation on X is successively performed in a similar manner. In this calculation for the exponential integral function $E_2(\tau)$

$$E_2(\tau) = \tau(\ln\tau + E - 1) - \sum_{\substack{m=0 \\ m\neq 1}}^{\infty} \frac{(-\tau)^m}{(m-1)m!} \quad (21)$$

is used, where E denotes Euler's constant.

4. EVALUATION RESULTS AND DISCUSSIONS

4.1 *Temperature profiles*

The typical results of temperature for both dispersed and fluid phases in Figs. 3–7, whose ordinate denotes the normalized dimensionless temperature as $\Theta = (\theta - \theta_0)/(\theta_w - \theta_0)$. There exist a couple of parameters which affect the temperature profiles and heat transfer in flowing multiphase media and further, the crucial interactions between these parameters need the speculative considerations. The pronounced features on the temperature profiles are as follows, that is, the temperatures for both phases around the central core in a duct increase appreciably due to the radiative transfer to the cloud of particles, provided that the optical length from the heating wall is appropriate, while the temperatures decrease in the vicinity of wall because of the presence of particles which might be presumably attributed not only to the comparatively large heat capacity of particles but to the absence of conductive heat transmission through particulate phase. The latter effect depicts the temperature gradient at the wall steeper and facilitates the convective heat transfer and the former increases the cup-mixing mean temperature. It might be expected that the heat transfer characteristic shows a drastic increase owing to the cumulative effects (increment of cup-mixing mean temperature by thermal radiation and that of the temperature gradient at the wall). These effects are prominent behaviours of heat transfer with thermal radiation in multiphase flow and show a remarkable contrast with radiative heat transfer in a single phase absorbing and emitting medium where the temperature gradient at the wall tends to decrease owing to the temperature rise of medium in an entire range of cross section by thermal radiation [10].

Figure 3 illustrates the temperature profiles of two phases with the variation of parameter X.

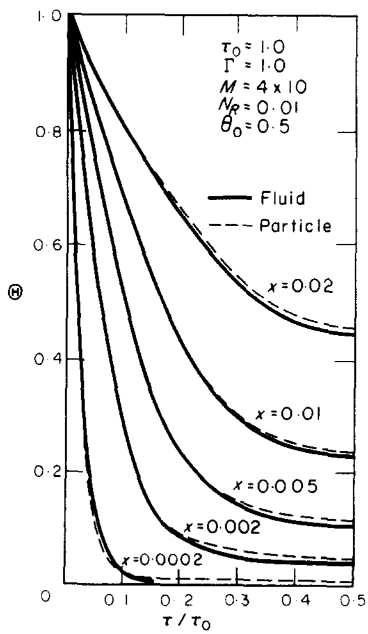


FIG. 3. Temperature profile vs. τ/τ_0 (effect of X).

One may rewrite the parameter X

$$X = (x/y_0)/(RePr = (x/y_0)B_0N_R/\tau_0 \quad (22)$$

where B_0 denotes the Boltzmann number which is the ratio of radiative heat transfer to convective and defined as

$$B_0 = 4\sigma T_w^3/\rho_f c_f \bar{u}_f. \quad (23)$$

Therefore one can examine this illustration from an alternative viewpoint. The temperature profiles show, in general, the substantial difference from those of Graetz's solution [11], where the thermal radiation is not taken into account, and also presents some interesting features which will be examined in the following. The temperature difference between particulate and fluid phase, which depends on the dimensionless parameter M (where M is related to the heat transfer between two phases), tends to decrease as X is increased in the immediate vicinity of wall ($\theta^f > \theta^p$), while the variation of the temperature difference in the central core of duct ($\theta^f < \theta^p$) is seemingly tranquil. Alternatively the location $(\tau/\tau_0)_e$, where the equality of temperatures for both phases is realized, is situated in the vicinity of the wall and thereby the temperature of particles is higher than that of fluid in the major portion of entire cross section. The location of $(\tau/\tau_0)_e$ has a trend of slight increase as X is increased up to certain value and, thereafter, has an inversed trend as X is further increased. For the cup-mixing mean temperatures of both phases the mean temperature of dispersed phase is higher than that of fluid throughout the duct from the very entrance edge with an exception of the large value of N_R (conduction predominates over radiation). The difference of mean temperature between both phases tends to increase gradually up to a certain difference and afterward tends to decrease as X increases.

The effects of N_R on temperature profiles are shown in Fig. 4 (a) and (b) for two values of X (= 0·0004, 0·01). At X = 0·0004 (near the entrance or at high Reynolds number) the location of $(\tau/\tau_0)_e$ approaches to the heating

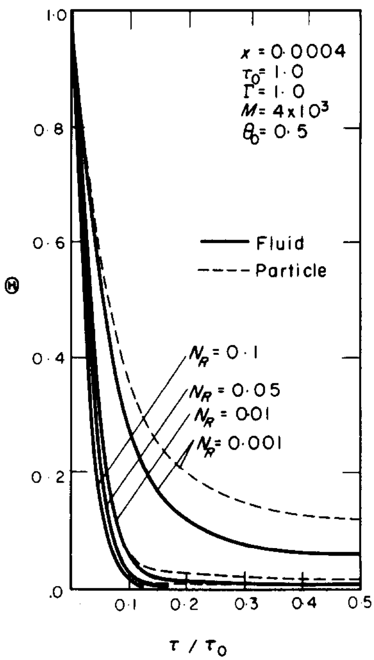


FIG. 4(a). Temperature profile vs. τ/τ_0 (effect of N_R).

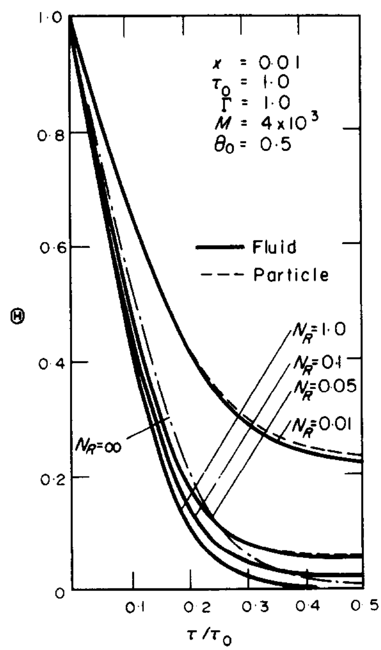


FIG. 4(b). Temperature profile vs. τ/τ_0 (effect of N_R).

wall as N_R decrease (radiation predominates over conduction) and at $X = 0{\cdot}01$ (far apart from the entrance or at low Reynolds number) the range where the fluid temperature is higher than that of particles penetrates to the central part in a duct in case of large value of N_R and, in consequence, the mean temperature of fluid may happen to be higher than that of particles. On the contrary the temperature of particulate phase yields a remarkable excursion from that of fluid at very small value of N_R where the multiphase flow is exposed in an extremely high temperature heat transfer (for example $N_R = 0{\cdot}001$ in Fig. 4(a)). The chain line in Fig. 4 is related to the temperature profile of single phase without thermal radiation. Figures 5 (a) and (b) concern the effects of the optical thicknesses of a duct on the temperature profiles. In case of large optical thicknesses the temperatures at the duct centre are slightly changed with the variation of the optical thickness, while the temperature through a wide range in duct

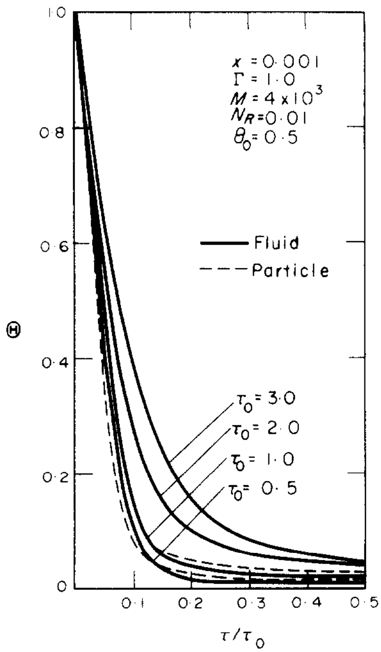


FIG. 5(b). Temperature profile vs. τ/τ_0 (effect of τ_0).

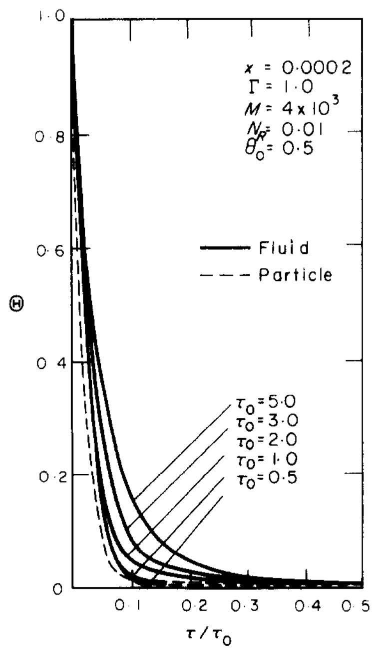


FIG. 5(a). Temperature profile vs. τ/τ_0 (effect of τ_0).

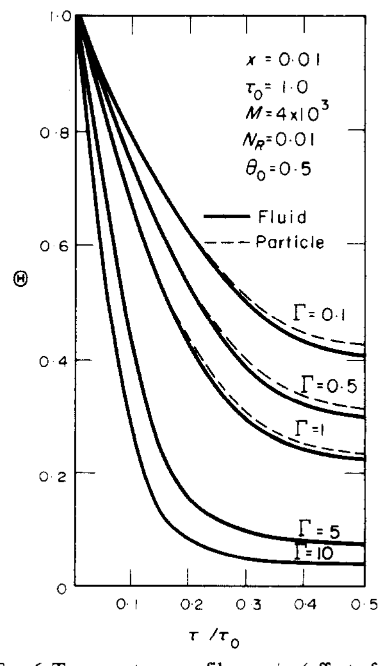


FIG. 6. Temperature profile vs. τ/τ_0 (effect of Γ).

is increased appreciably with the optical thickness. The other feature is that the temperature differences between two phases become indistinguishable for the optical thickness $\tau_0 \geqq 2$. The influences of the thermal loading ratio are depicted in Fig. 6, in which the temperatures of both media with low thermal loading ratio indicate much higher than those with high thermal loading ratio due to the large heat capacity of flowing medium and simultaneously the temperature of particles becomes rather high compared with that of fluid at low thermal loading ratio. From a practical viewpoint it is noted that the multiphase flow with high particle load is plausible regime for the heat transfer of high heat fluxes but not suitable in order to get high temperature gases at an exit. Finally Fig. 7 shows the temperature profiles

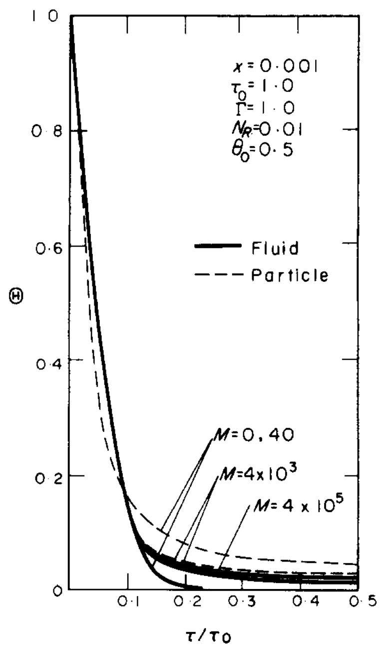


FIG. 7. Temperature profile vs. τ/τ_0 (effect of M).

when the dimensionless parameter M is varied. The example of $M = 4 \times 10^3$ corresponds to the case of $Nu_d \simeq 2$ for $d_p \simeq 10\mu$, $Nu_d \simeq 20$ for $d_p \simeq 10^2\mu$ and $Nu_d \simeq 200$ for $d_p \simeq 10^3\mu$ with the parameters of $\tau_0 = 1$, $\varepsilon_p = 1$ and $\kappa \sim 0(1)$.

For the actual calculations $Y_0/2$ is divided into 25 equal intervals and the lattice interval along X near the entrance is $\frac{1}{2}(\Delta Y)^2$ and the interval is gradually increased. It can be easily understood from the factor (τ_0^2/N_R) in equation (18) that the convergence of solution requires a number of iterative calculations for large τ_0 and small N_R and further, that the convergence is not fulfilled for the extreme cases. It may be noted that one can resolve this difficulty by replacing a small interval of ΔX.

4.2 *Heat transfer*

The overall heat flux at the wall q_{total} is a sum of the conductive heat flux q_{con} and the radiative heat flux q_{rad}

$$q_{\text{total}} = q_{\text{con}} + q_{\text{rad}}. \tag{24}$$

The local Nusselt number Nu_x at the length x from entrance is defined as

$$Nu_x = \frac{h_x d_e}{k_f} = \frac{q_{\text{total}} d_e}{k_f(T_w - T_m)} = Nu_{x,\text{con}} + Nu_{x,\text{rad}} \tag{25}$$

where d_e denotes the equivalent hydrodynamic diameter and q_{con}, q_{rad}, $Nu_{x,\text{con}}$ and $Nu_{x,\text{rad}}$ are represented in the forms

$$q_{\text{con}} = -k_f\left(\frac{\partial T}{\delta y}\right)_{y=0} \tag{26}$$

$$q_{\text{rad}} = [R(0)E_3(0) - R(\tau_0)E_3(\tau_0) - \int_0^{\tau_0} E_{bb}(\tau)E_2(\tau)\mathrm{d}\tau] \tag{27}$$

$$Nu_{x,\text{con}} = q_{\text{con}} \cdot d_e/k_f(T_w - T_m) \tag{28}$$

$$Nu_{x,\text{rad}} = q_{\text{rad}} \cdot d_e/k_f(T_w - T_m). \tag{29}$$

The cup-mixing mean temperatures for fluid, particulate and multiphase media in dimensionless forms are

$$\theta_{m,f} = \int_0^1 U\theta_f \mathrm{d}Y \tag{30}$$

$$\theta_{m,p} = \int_0^1 U\theta_p \mathrm{d}Y \tag{31}$$

$$\theta_m = \int_0^1 \{\theta_f + \Gamma\theta_p\}U\,dY/\{1+\Gamma\}. \quad (32)$$

Substituting equations (26) and (32) into equation (28) and equations (27) and (32) into equation (29) one gets

$$Nu_{x,\text{con}} = -\left(\frac{\partial\theta_f}{\partial Y}\right)_{Y=0}\Bigg/(1-\theta_m) \quad (33)$$

$$Nu_{x,\text{rad}} = \frac{\tau_0}{2N_R(1-\theta_m)}$$

$$[E_3(0) - E_3(\tau_0) - \tau_0\int_0^1 \theta^4(Y)E_2(\tau_0 Y)dY]. \quad (34)$$

Substitution of equation (30) instead of equation (32) into equation (28) yields another Nusselt number

$$Nu'_{x,\text{con}} = -\left(\frac{\partial\theta^f}{\partial Y}\right)_{Y=0}\Bigg/(1-\theta_{m,f}). \quad (35)$$

Although the physical meaning of the Nusselt number defined in equation (25) is prevalent and valid, the meanings of Nusselt numbers in equations (33) to (35) are eventually ambigious. In a strict sense of Nusselt number (for example equation (25)) the mean temperature has to be correlated to the local heat flux and its history along x. Noting on the dispersed phase the heat quantity which is transferred to this phase consists not only of the heat quantity substracting that transferred to fluid from the radiative transferred energy to particles for the region

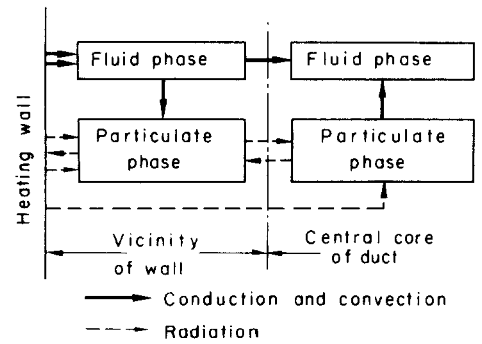


FIG. 8. Heat transfer mechanism.

where the temperature of particles exceeds but also of the heat which is transferred from fluid to particles in the vicinity of the wall. For the fluid phase the net quantity of heat transferred to this phase can be found by a similar argument for the dispersed phase. The typical heat transfer mechanism of multiphase flow is schematically sketched in Fig. 8. Consequently in the rigorous definition of Nusselt number which evaluates the heat transfer characteristics for each phase the substantial heat flux to an alternative medium has to be accounted coupling with the mean temperature of the corresponding medium. In this paper equation (35) is tentatively used for the comparison to the Nusselt number of convective heat transfer without radiation and shown as a family of dotted curves in Fig. 9.

Figure 9 illustrates the relations of Nu_x and Nu'_x vs. X as parameters of radiative $\sim$ conductive interaction N_R, optical thickness τ_0 and thermal loading ratio of particle Γ. The behaviour of Nu_x against X, that Nu_x has a certain minimum and tends to increase following this point, is peculiar to combined heat transfer with radiation which means that the temperature field in flowing medium is not fully developed in a down stream. The interaction between radiation and convection in multiphase flow seems to be rather strong compared with that in single phase flow of radiation absorbing and emitting medium. Reference to the figure indicates that the influences of N_R and τ_0 on Nu_x are drastic and that of Γ is seemingly small while Γ affects Nu'_x appreciably. The preceding discussions reveal the heat transfer mechanism in multiphase flow to some extent and further elaborate examination will be performed in the subsequent studies. In Fig. 10 the cup-mixing mean temperatures of multiphase are plotted against X. The dotted line corresponds to the single phase ($\Gamma = 0$) without radiation ($N_R = \infty$). θ_m for $N_R = 1{\cdot}0$ and $0{\cdot}1$ is lower than the dotted line because the mass flow rate (or heat capacity) of medium ($\Gamma = 1$) is about two times of single phase medium ($\Gamma = 0$).

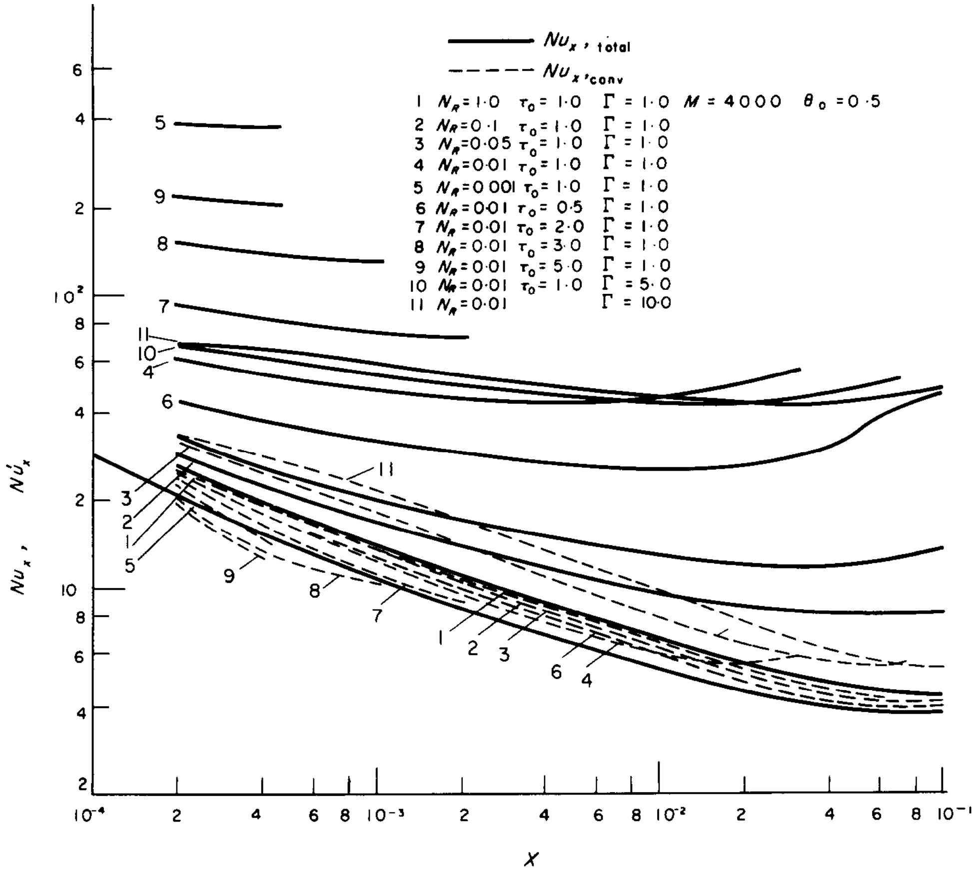


FIG. 9. Nusselt numbers Nu_x, $N'u_x$ vs. X.

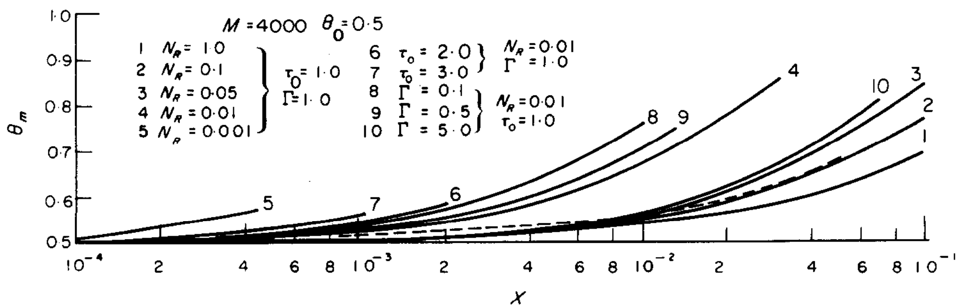


FIG. 10. Mean temperature of multiphase medium vs. X.

4.3 Relations of the dimensionless parameters and discussions

Among the dimensionless parameters used in present study X, N_R and τ_0 are familiar to the radiative heat transfer. The optical thickness τ_0 is, however, correlated to the particulate system whilst the absorption coefficient κ is a physical property in a radiation absorbing and emitting medium. Consequently some brief discussions will be developed on the optical thickness, thermal loading ratio of particles and heat transfer parameter between two phases. The algebraic manipulation of κ leads to

$$\kappa = \varepsilon_p \pi \left(\frac{d_p}{2}\right)^2 n_p = \left(\frac{\rho_f c_f}{\rho_p c_p}\varepsilon_p\right)\left(\frac{\bar{u}_f/\bar{u}_p}{d_p/2}\Gamma\right) \tag{36}$$

where κ is related to Γ. The familiar parameters such as X and N_R contain the absorption coefficient κ as

$$X = \frac{x/y_0}{PrRe} = \frac{\kappa x}{\tau_0^2} N_R B_0 \tag{37}$$

$$N_R = k_f \varkappa / 4n^2 \sigma T_w^3. \tag{38}$$

Then one has to examine the effects of these parameters in the heat transfer system of multiphase flow. By the way the first factor in equation (36) concerns to the physical properties of multiphase media and the second to the flow system. The relations among the parameters of τ_0, M and Γ are

$$\tau_0 = \left(\frac{\rho_f c_f}{\rho_p c_p}\varepsilon_p\right)\left(\frac{y_0}{d_p/2}\Gamma\right) \tag{39}$$

$$M = \left(\frac{\rho_f c_f}{\rho_p c_p}\varepsilon_p\right)^{-1} \frac{8}{3} Nu_d \frac{\bar{u}_p}{\bar{u}_f}\frac{\tau_0^2}{\Gamma} = \left(\frac{\rho_f c_f}{\rho_p c_p}\right)\frac{8}{3}\frac{\bar{u}_f}{\bar{u}_p}\left(\frac{y_0}{d_p/2}\right)^2 \Gamma. \tag{40}$$

Similarly the first factors in equations (39) and (40) are referred to the physical properties of both phases. Helium, nitrogen and air for fluid and water, magnesium, carbon and aluminium for particulate phase will be utilized for the practical purposes. In these illustrations $\rho_f/\rho_p = 0(10^{-3}) \sim 0(10^{-4})$. Equation (39) indicates that τ_0 is proportional to y_0 and Γ, and inversely proportional to d_p. In order to change the optical thickness τ_0, which is of great significance in heat transfer characteristics, it does not necessitate to pressurise the whole loop of circulation as in case of a radiating gas but it is feasible to realize by varying the loading of particles and the particle's diameter. This fact leads to a particular interest in engineering applications.

An examination of equation (40) shows that M is proportional to $(Nu_d . d_p^{-2})$ under constant Γ. In other words for small particles M is large which reduces the temperature difference between two phases and vice versa. In an extremely high temperature system, however, the temperature difference might be appreciable unless the augmentation on heat transfer between particles and fluid in better perspective by means of the turbulence producer, exerting the body forces by electromagnetic or sonic vibration and so forth.

Finally though these parameters encountered in this system are coupled together it must be emphasized that one may be able to alter the parameter independently by changing the combination of two media, particle's diameter, number density of particles and so on and then the desired situation might be in reality, while in a radiation absorbing and emitting gas it is often formidable to realize a plausible physical condition which is predicted theoretically.

5. CONCLUDING REMARKS

An analysis has been performed on the radiative heat transfer by flowing multiphase medium at high or extremely high temperatures. The physical model employed here is based on laminar flow in parallel flat plates and yields the following conclusions.

(1) On the temperature profiles of multiphase medium the temperature gradient at the wall becomes steeper while the cup-mixing mean temperature is increased by the temperature

rise at the central core in a duct due to the radiative heating of particulate medium and these effects lead to the high heat transfer characteristics.

(2) The effects of dimensionless parameter group (X, N_R, τ_0, Γ and M) on temperature profiles, heat transfer characteristics and cup-mixing mean temperature are elucidated systematically.

(3) The examination of heat transfer mechanism in multiphase flow and the discussions on the analytical results show that this system is plausible means for high temperature and high flux heat transfer and promising for engineering applications to a great extent.

REFERENCES

1. L. FARBAR and M. J. MORLEY, Heat transfer to flowing gas–solids mixtures in a circular tube, *Ind. Engng Chem.* **49**,1143 (1957).
2. C. L. TIEN, Heat transfer by a turbulently flowing fluid–solids mixture in a pipe, *J. Heat Transfer* **83**, 183 (1961).
3. C. A. DEPEW and L. FARBER, Heat transfer to pneumatically conveyed glass particles of fixed size, *J. Heat Transfer* **85C**, 164 (1963).
4. S. L. SOO, *Fluid Dynamics of Multiphase Systems.* Blaisdell, Massachusetts (1967).
5. R. G. BOOTHROYD, Pressure drop in duct flow of gaseous suspensions of fine particles, *Trans. Instn Chem. Engrs.* **44**, 306 (1966).
6. D. C. SCHLUDERBERG, R. L. WHITELAW and R. W. CARLSON, Gaseous suspensions—a new reactor coolant, *Nucleonics* **19** (8), 67 (1961).
7. V. S. NOSOV and N. I. SYROMYATNIKOV, Laws of heat loss for finely dispersed flows, *Sov. Phys. Dokl.* **10**, 672 (1966).
8. R. G. BOOTHROYD and H. HAQUE, Fully developed heat transfer to a gaseous suspension of particles flowing turbulently in ducts of different size, *J. Mech. Engng Sci.* **12** (3), 191 (1970).
9. R. VISKANTA, Radiation transfer and interaction of convection with radiation heat transfer, *Advances in Heat Transfer*, Vol. 3, p. 212. Academic Press, New York (1966).
10. Y. KUROSAKI, Heat transfer by radiation and other transport mechanisms (2nd Report, flow between parallel flat plates with simultaneous radiation and convection), *Proc. Japan Soc. Mech. Engrs*, **35** (278), 2099 (1969).
11. J. R. SELLARS, M. TRIBUS and J. S. KLEIN, Heat transfer to laminar flow in a round tube or flat conduit—the Graetz problem extended, *Trans. Am. Soc. Mech. Engrs* **78**, 441 (1956).

RAYONNEMENT THERMIQUE PAR UN MILIEU MULTIPHASIQUE EN ECOULEMENT. lère PARTIE: ANALYSE DU TRANSFERT THERMIQUE D'UN ECOULEMENT LAMINAIRE ENTRE DEUX PLAQUES PLANES PARALLELES

Résumé— L'écoulement multiphasique de suspensions gazeuses de fines particules assure de hautes caractéristiques de transfert thermique à des températures élevées et à des grands flux thermiques dus au transfert rayonnant depuis une source thermique vers les suspensions. L'aspect discontinu du milieu dispersé améliore remarquablement le transfert thermique global et du point de vue pratique il existe un prolongement important dans les applications industrielles.

Il est important de faire une étude plus précise des comportements des suspensions et du mécanisme de transfert thermique dans des milieux multiphasiques en écoulement de façon à conduire une discussion détaillée.

On a mené une analyse de l'écoulement laminaire entre des plans parallèles en tenant compte du rayonnement thermique. Les résultats illustrent les profils de température du fluide et de la phase dispersée et les caractéristiques du transfert thermique pour de larges domaines des paramètres sans dimension tels que le paramètre d'interaction conduction–rayonnement, le rapport de charge des particules, la profondeur optique de la conduite, le transfert thermique entre les deux phases. Un examen des profils de température révèle que tandis que le gradient de température au voisinage de la surface chauffée croît à cause de la présence de la phase dispersée, la température moyenne du mélange est augmentée de façon appréciable par le rayonnement thermique à travers le milieu dispersé. En conséquence, la contribution des suspensions sur le transfert thermique est importante, particulièrement dans les cas des températures élevées. Alternativement, les relations entre les paramètres sans dimension précédents sont aussi examinées dans cette étude.

WÄRMEÜBERTRAGUNG DURCH STRAHLUNG BEI EINEM FLIESSENDEN VIELPHASEN-MEDIUM. TEIL I: EINE UNTERSUCHUNG DES WÄRMEÜBERGANGS BEI LAMINARER STRÖMUNG ZWISCHEN PARALLELEN EBENEN PLATTEN

Zusammenfassung—Infolge des Wärmeübergangs durch Strahlung von der Wärmequelle auf suspendierte Teilchen, liefert die Vielphasenströmung von Gassuspensionen feiner Teilchen hohe Wärmestromcharakteristiken bei hohen und/oder sehr hohen Temperaturen und hohen Wärmeströmen. Die Phasenänderung des Teilchenmediums verbessert den gesamten Wärmeübergang erheblich. Praktisch ist dies für die industrielle Anwendung wichtig.

Es lohnt, das Verhalten der Suspensionen und den Mechanismus in strömenden Mehrphasenmedien näher zu betrachten, um eingehendere Diskussionen anzuregen.

Die laminare Strömung zwischen parallelen Platten wurde untersucht, wobei die Temperaturstrahlung berücksichtigt wurde. Die Ergebnisse geben die Temperaturprofile des Fluids bzw. der dispergierten Phase wieder und Wärmeübergangscharakteristiken für einen weiten Bereich von dimensionslosen Parametern wie den Wechselwirkungsparameter Strahlung-Leitung, Anteil der Teilchen, optische Breite des Kanals, Wärmeübergang zwischen den beiden Phasen u.s.w.

Die Temperaturprofile ergeben: während der Temperaturgradient in der Nachbarschaft der Heizfläche ansteigt, aufgrund der Anwesenheit der Teilchenphase, nimmt die mittlere Mischungstemperatur wegen der Temperaturstrahlung durch das dispergierte Medium hindurch merklich zu. Folglich sind die Beiträge von Suspensionen zum Wärmeübergang gross, besonders im Falle hoher Temperaturen. Ausserdem werden in der vorliegenden Arbeit die Beziehungen zwischen den oben erwähnten dimensionslosen Parametern geprüft.

ЛУЧИСТЫЙ ПЕРЕНОС ПРИ ТЕЧЕНИИ МНОГОФАЗНОЙ СРЕДЫ

ЧАСТЬ 1. Анализ теплообмена в ламинарном потоке между плоскопараллельными пластинами

Аннотация—Многофазный поток газовзвеси мелких частиц характеризуется высокоинтенсивным теплообменом при высоких или сверхвысоких температурах и больших тепловых потоках благодаря лучистому переносу от источника тепла к взвесям. Относительное перемещение фаз дисперсной среды значительно улучшает общий теплообмен, что с практической точки зрения очень важно для промышленного применения.

Особое внимание уделяется поведению сусензий и механизму теплообмена при течении многофазных сред.

Анализируется ламинарное течение между параллельными пластинами с учетом тепловой радиации. Приведены данные, описывающие распределение температуры жидкости и диспергированной фазы, а также теплообменные характеристики в широком диапазоне таких безразмерных параметров, как параметр взаимодействия излучения и теплопроводности, относительная концентрация частиц, оптическая толщина трубопровода, теплообмен между двумя фазами и т. д. Обращение к температурным профилям обнаруживает, что в то время, как градиент температур вблизи поверхности нагрева увеличивается благодаря наличию дисперсной фазы, средняя температура перемешивания значительно повышается в результате теплового излучения через дисперсную среду. Вследствие этого, влияние взвесей на теплообмен очень велико, в особенности, при высоких температурах. Рассматриваются также соотношения между приведенными безразмерными параметрами.